

Inverse

Exercise 2.13

a. $A = \begin{pmatrix} -1 & i \\ 1+i & 0 \end{pmatrix}$

b. $B = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

c. $C = \begin{pmatrix} 1 & -1 & i \\ 1+i & 0 & \end{pmatrix}$

Solution Exercise 2.13

a.
$$A = \begin{pmatrix} -1 & i \\ 1+i & 0 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} -1 & i & 1 & 0 \\ 1+i & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{\frac{1}{2}-\frac{1}{2}iR_2 \leftrightarrow R_1} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ -1 & i & 1 & 0 \end{array} \right) \xrightarrow[\sim]{1R_1 \rightarrow R_2} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ 0 & i & 1 & \frac{1}{2}-\frac{1}{2}i \end{array} \right) \xrightarrow[\sim]{-1iR_2}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ 0 & 1 & -1 & i-\frac{1}{2}-\frac{1}{2}i \end{array} \right)$$

$$\text{inv } A = \begin{pmatrix} 0 & \frac{1}{2}-\frac{1}{2}i \\ -1 & i-\frac{1}{2}-\frac{1}{2}i \end{pmatrix}$$

b.
$$B = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 1 & i & 1 & 0 \\ i & 1 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{-1iR_2 \leftrightarrow R_1} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 1 & i & 1 & 0 \end{array} \right) \xrightarrow[\sim]{-1R_1 \rightarrow R_2} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 0 & 2i & 1 & i \end{array} \right) \xrightarrow[\sim]{-\frac{1}{2}iR_2} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 0 & 1 & -\frac{1}{2}i & \frac{1}{2} \end{array} \right) \xrightarrow[\sim]{iR_2 \rightarrow R_1}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 0 & 1 & -\frac{1}{2}i & \frac{1}{2} \end{array} \right)$$

$$\text{inv } B = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}i \\ -\frac{1}{2}i & \frac{1}{2} \end{pmatrix}$$

c.

$$C = \begin{pmatrix} 1 & -1 & i \\ 1+i & 0 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|cc} 1 & -1 & i & 1 & 0 \\ 1+i & 0 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{\frac{1}{2}-\frac{1}{2}iR_2 \leftrightarrow R_1} \left(\begin{array}{ccc|cc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 1 & -1 & i & 1 & 0 \end{array} \right) \xrightarrow[\sim]{-1R_1 \rightarrow R_2} \left(\begin{array}{ccc|cc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 0 & -1 & i & 1 & -\frac{1}{2} + \frac{1}{2}i \end{array} \right) \xrightarrow[\sim]{iR_2}$$

$$\left(\begin{array}{ccc|cc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 0 & 1 & i & -\frac{1}{2} & -\frac{1}{2}i \end{array} \right)$$

$$\text{inv } C = \begin{pmatrix} 0 & \frac{1}{2} - \frac{1}{2}i \\ i & -\frac{1}{2} - \frac{1}{2}i \end{pmatrix}$$